1. Consider a market where there are two consumers with inverse demand functions \( p(q_1) = 10 - q_1 \) and \( p(q_2) = 5 - q_2 \).

(a) Suppose there is a single firm with inverse supply function \( p(q) = \frac{1}{2} q \). Find the competitive equilibrium.

Solution: Adding the two individual demands yield

\[
q_1 = 10 - p \\
q_2 = 5 - p
\]

\[
\Rightarrow Q^D = \begin{cases} 
10 - p & \text{if } 5 \leq p \leq 10 \\
15 - 2p & \text{if } p \leq 5
\end{cases}
\]

Since there is only one firm, the market supply function is \( Q^s = 2p \). The equilibrium condition \( Q^D = Q^S \) has no solution when \( 5 \leq p \leq 10 \). So, using the portion of \( Q^D \) below \( p \leq 5 \) yields:

\[
Q^D = Q^S \\
15 - 2p = 2p \\
\Rightarrow p^* = 3.75 \\
Q^* = 2(3.75) = 7.5.
\]

(b) Find the elasticity of demand and supply at the equilibrium.

Solution: The two elasticities are

\[
\varepsilon_D = \frac{\partial Q^D}{\partial P} \left( \frac{P}{Q^D} \right) \\
= (-2) \left( \frac{3.75}{7.5} \right) = -1,
\]

and

\[
\eta_D = \frac{\partial Q^S}{\partial P} \left( \frac{P}{Q^S} \right) \\
= (2) \left( \frac{3.75}{7.5} \right) = 1
\]

(c) Suppose instead that there are three firms with the identical inverse supply function given in part (a). Find the competitive equilibrium.
Solution: Now, $Q^s = 6p$. Again, $Q^D = Q^S$ has no solution when $5 \leq p \leq 10$. Using the portion of $Q^D$ below $p \leq 5$ yields:

\[
Q^D = Q^S \\
15 - 2p = 6p \\
\Rightarrow p^* = 1.875 \\
Q^* = 6(1.875) = 11.25.
\]

2. Consider a government that wants to raise revenue by implementing a per unit tax of $t$ on a commodity. The government has a choice of taxing one of two markets. Both markets have the usual downward-sloping, linear demand curves. However, at the current equilibrium (that is, before taxation), the demand in market 1 is inelastic while in market 2 it is elastic. To keep the situation simple, assume that the two markets have exactly the same supply function, which is perfectly elastic at price $\bar{p}$. Assume further that the equilibrium quantity prior to taxation is also the same in both markets.

(a) What are possible reasons for taxing market 1 rather than market 2?

Solution: As seen in the graph, more inelastic demand implies less quantity reduction by the consumers. So, the tax revenue will be higher. In the graph below, revenue from taxing market 1 is $A + B$ while revenue from market 2 is only $A$.

(b) What are possible reasons for taxing market 2 rather than market 1?

Solution: More tax revenue necessarily in this example means greater burden on the consumer.
(c) In light of your answers above, should government tax cigarettes?

**Solution:** Any serious answer will do. Here are some possibilities. Since demand for cigarette is inelastic, taxing cigarette may be an effective way to generate revenue. However, cigarette demand is inelastic because of its addictive effect. Therefore, taxing this market may be viewed as an exploitation of the unfortunate addicts. Of course, one may wish to incorporate the health dimension and the associated cost of cigarette smoking into the discussion.

3. Consider a market where the supply is given by \( Q^S = P \) and the demand is given by \( Q^D = 20 - P \).

(a) Suppose the government wants to raise 18¥ by imposing per unit tax on this market. What tax rate will raise the required revenue and also minimize the dead weight loss?

**Solution:**

Letting \( P^D = P^S + t \), we obtain

\[
Q^D = 20 - (P^S + t) = P^S = Q^S
\]

\[\Rightarrow P^S = \frac{20 - t}{2}\]

\[Q^S(t) = \frac{20 - t}{2}\]

Setting \( 18 = GR = tQ^s(t) = t\left(\frac{20 - t}{2}\right) \),

yields \( t^2 - 20t + 36 = 0 \)

\[ (t - 2)(t - 18) = 0 \]

\[\Rightarrow t = 2 \text{ or } 18.\]
Since
\[ Q^s(2) = 9 > 1 = Q^s(18), \]
t = 2 will minimize the dead weight loss.

(b) What is the resulting equilibrium and the dead weight loss? What is the incidence of taxation?

**Solution:** Quantity traded is \( Q^S = 9, P^S = 9, \) and \( P^D = 11. \) The dead weight loss is \( DWL = (0.5)(2)(10 - 9) = 1, \) and the incidence of tax is \( \frac{11 - 10}{2} = 0.5 \) for the consumer and \( \frac{10 - 9}{2} = 0.5 \) for the producer.

(c) Can you think of a method for raising 18¥ from this market that will (1) incur no dead weight loss and (2) be preferable to the per unit tax for both the producers and the consumers.

**Solution:** Since the tax burden is split equally, the government can simply demand lump-sum tax of 9¥ from the producers and 9¥ from the consumers. Since quantity traded remains the same as the competitive equilibrium, this will result in no dead weight loss. Hence, both the producers and the consumers will be better off under this scheme than the per unit tax.

4. Consider a market where the supply is given by \( Q^S = P - 2 \) and the demand is given by \( Q^D = 10 - P. \)

(a) Find the competitive equilibrium. What is the consumer, the producer, and the aggregate surplus?

**Solution:**
Competitive equilibrium is found by:

\[ Q^D = 10 - P = P - 2 = Q^S \]
\[ \Rightarrow P^* = 6 \]
\[ Q^* = 4. \]
\[ CS^* = (0.5)(10 - 6)(4) = 8 \]
\[ PS^* = (0.5)(6 - 2)(4) = 8. \]

(b) Suppose the government wants to encourage production by instituting a subsidy of 2¥ per unit. What is the impact of the subsidy on the quantity traded, the prices, the consumer surplus and the producer surplus?

**Solution:** Subsidy works like negative tax. That is \( P^D = P^S - s \), where \( s \) denotes the subsidy. Thus, we have

\[ Q^D = 10 - (P^S - 2) = P^S - 2 = Q^S \]
\[ \Rightarrow P^S = 7 \]
\[ P^D = 5 \]
\[ \hat{Q} = 5. \]
\[ \hat{CS} = (0.5)(10 - 5)(5) = 12.5 \]
\[ \hat{PS} = (0.5)(7 - 2)(5) = 12.5 \]

(c) Suppose the subsidy the government pays will have to be raised by levying lump-sum tax on the consumers. What is the impact of the subsidy on the consumer’s welfare? What is the impact on the welfare if the tax burden is shared equally by the consumers and the producers?

**Solution:** Government revenue is \(-(s)(\hat{Q}) = -2(5) = -10\). If the entire amount comes from the consumers, their total surplus is now

\[ \hat{CS} = 12.5 - 10 = 2.5 < 8 = CS^*. \]

If the burden is shared equally, then

\[ \hat{CS} = 12.5 - 5 = 7.5 < 8 = CS^* \]
\[ \hat{PS} = 12.5 - 5 = 7.5 < 8 = PS^*. \]

Note that the deadweight loss from the subsidy is -1. One way or the other, the burden must be born by the consumers and/or the producers.

5. Consider a market where the domestic supply and the domestic demand are given by

\[ Q^S(P) = 200P \quad \text{and} \quad Q^D(P) = 4000 - 200P. \]
(a) Suppose this is a closed economy. Find the equilibrium price and quantity. What are the consumer surplus, producer surplus, and aggregate surplus?

**Solution:** Setting \( Q^s = Q^D \) yields

\[
200P = 4000 - 200P
\]

\[
P^* = \frac{4000}{400} = 10
\]

\[
\Rightarrow Q^* = 200(10) = 2000.
\]

Inverse supply and demand functions are:

\[
P = \frac{Q^S}{200} \quad \text{and} \quad P = 20 - \frac{Q^D}{200}.
\]

\[
\Rightarrow CS^* = 0.5(20 - 10)(2000) = 10,000
\]

\[
PS^* = 0.5(10 - 0)(2000) = 10,000
\]

\[
AS^* = CS^* + PS^* = 20,000.
\]

For the remainder of the question, assume that the economy is open.

(b) The world supply is perfectly elastic at price \( P_w = 5 \). Find the equilibrium price, quantity traded, and total import. What are the consumer surplus, producer surplus, aggregate surplus, and gains from the trade?

**Solution:** Since the world supply is perfectly elastic at \( P_w < P^* \), the equilibrium price will fall to \( P_w = 5 \).

Quantity traded = \( Q^D(P_w) = 4000 - 200(5) = 3,000 \)

Domestic supply = \( Q^S(P_w) = 200(5) = 1,000 \)

Import = \( Q^D(P_w) - Q^S(P_w) = 3000 - 1000 = 2,000 \)

\[
CS^o = 0.5(20 - 5)(3000) = 22,500
\]

\[
PS^o = 0.5(5 - 0)(1000) = 2,500
\]

\[
AS^o = 25,000
\]

Gains from the trade = \( AS^o - AS^* = 5,000 \).

(c) Suppose the government wants to help the domestic producers by limiting the imports to 1,200 units. Find the domestic equilibrium price, quantity traded, and supply. Assuming that the price of the imported goods will rise to the same level as the domestically produced goods, find the consumer surplus, producer surplus and aggregate surplus. What is the dead weight loss compared to free trade?
Solution: Import = $Q^D - Q^S$. So, if imports are restricted to 1,200 units, domestic price will rise until

$$Q^D(P) - Q^S(P) = 4000 - 200P - 200P = 1,200$$

$$P^q = \frac{2800}{400} = 7$$

Quantity traded = $Q^D(P^q) = 4000 - 200(7) = 2,600$

Domestic supply = $Q^S(P^q) = 200(7) = 1,400$

Import = 1,200

$$CS^q = 0.5(20 - 7)(2600) = 16,900$$

$$PS^q = 0.5(7 - 0)(1400) = 4,900$$

$$AS^q = 21,800$$

$$DWL^q = 3,200.$$

(d) Suppose the government wants to reduce the imports to 1,200 units by using tariffs rather than direct quota. How should the government set the tariff to achieve this? Compare the dead weight loss under tariff and quota.

Solution: From Part (c), we see that the effective price has to be $p^q = 7$ to reduce the imports to 1200. Therefore, tariff needs to be $t = 2$.

$$CS^t = 0.5(20 - 7)(2600) = 16,900$$

$$PS^t = 0.5(7 - 0)(1400) = 4,900$$

$$GR^t = (\text{tariff})(\text{import}) = 2(1200) = 2,400$$

$$AS^t = 21800 + 2400 = 24,200$$

$$DWL^t = 800.$$

Since the effect of the import tariff is identical to the quota, except that government gets to collects tax revenue, it results in smaller dead weight loss.

(e) Can you think of reasons why a government may want to use quota rather than tariff?

Solution: Any reasonable answer will do. Here’s one possibility. In practice, it’s difficult to measure how effective import tariff is in reducing foreign competition. In contrast, quota system gives the government direct control over the level imports. Therefore, if the government wants to be certain about the degree to which it is protecting the domestic producers, it may wish to use quota despite the higher social cost.
6. Consider again the market where the domestic supply and the domestic demand are given by

\[ Q^S(P) = 200P \quad \text{and} \quad Q^D(P) = 4000 - 200P. \]

(a) Suppose, now the world price is \( P_w = 12 \), which is above the closed economy equilibrium price, \( P^* \). As usual, assume that the world demand is perfectly elastic at \( P_w = 12 \) and that when indifferent between selling in the domestic market and the foreign market, producers sell in the domestic market first. Find the equilibrium price, quantity traded domestically, and total export. What are the consumer surplus, the producer surplus, and the aggregate surplus. Who gains and/or loses from trading?

**Solution:** Since \( P_w > P^* \) domestic producers will sell to the domestic consumers at price \( P_w \) and sell the remainder in the foreign market

\[
\begin{align*}
\text{Quantity traded domestically} & = Q^D(P_w) = 4000 - 200(12) = 1600 \\
\text{Domestic supply} & = Q^S(P_w) = 200(12) = 2400 \\
\text{Export} & = Q^S(P_w) - Q^D(P_w) = 2400 - 1600 = 800 \\
\text{CS}^o & = 0.5(20 - 12)(1600) = 6400 \\
\text{PS}^o & = 0.5(12 - 0)(2400) = 14400 \\
\text{AS}^o & = 20,800 \\
\text{Gains from the trade} & = \text{AS}^o - \text{AS}^* = 20800 - 20000 = 800.
\end{align*}
\]

When the economy is opened, the domestic price is raised to the world price. As a result, domestic consumers loses \( CS^* - CS^o = 10000 - 6400 = 3600 \) while producers gain \( PS^o - PS^* = 14400 - 1000 = 4400 \). Since producer’s gains are greater than the consumer’s loss, there is a net gain in the aggregate surplus of \(-3600 + 4400 = 800\).

(b) Suppose the government wants to encourage export by giving a subsidy of \( s = 1 \) per unit of export. Find the equilibrium price, quantity traded, and total import. What are the consumer surplus, the producer surplus, and the dead weight loss? Who wins and loses from the subsidy?

**Solution:** Subsidy raises the price that the producer gets from exporting to \( P^s = P_w + s = 13 \). Thus, domestic producers try to sell to the domestic consumers at price \( P^s \) and sell the remainder in the foreign market. However, whether the domestic consumer will buy the good at \( P^s \) depends on whether they have the option to buy the import at \( P_w \).
If they cannot buy imports, then they will be forced to accept $P^s$ so that

\[
\text{Quantity traded domestically} = Q^D(P^s) = 4000 - 200(13) = 1,400
\]
\[
\text{Domestic supply} = Q^S(P^s) = 200(13) = 2,600
\]
\[
\text{Export} = Q^S(P^s) - Q^D(P^s) = 2600 - 1400 = 1,200
\]
\[
CS^s = 0.5(20 - 13)(1400) = 4,900
\]
\[
PS^s = 0.5(13 - 0)(2600) = 16,900
\]
\[
GR^s = -s(\text{export}) = -(1)(1200) = -1,200
\]
\[
AS^s = 20,600
\]
\[
DWL^s = AS^s - AS^o = 20800 - 20600 = 200.
\]

As seen above, the subsidy increases the producer's surplus at the further expense of the consumers.

But, if the consumers can buy imports, then they will buy imports at price $P_w$ rather than the domestically produced goods at $P^s$. Therefore, domestic producer will sell all of its output in the foreign market.

\[
\text{Quantity traded domestically} = Q^D(P_w) = 4000 - 200(12) = 1,600
\]
\[
\text{Domestic supply} = Q^S(P^s) = 200(13) = 2,600
\]
\[
\text{Export} = Q^S(P^s) - 0 = 2600 = 2,600
\]
\[
CS^s = 0.5(20 - 12)(1600) = 6,400
\]
\[
PS^s = 0.5(13 - 0)(2600) = 16,900
\]
\[
GR^s = -s(\text{export}) = -(1)(2600) = -2,600
\]
\[
AS^s = 6400 + 16900 - 2600 = 20,700
\]
\[
DWL^s = AS^s - AS^o = 20800 - 20700 = 100.
\]

7. Consider a market where the domestic supply and the domestic demand are given by

\[Q^S(P) = 100P\quad\text{and}\quad Q^D(P) = 2000 - 100P.\]

Assume that the economy is open and that the world supply and the world demand is perfectly elastic at price $P_w = 5$.

(a) Find the equilibrium price and the quantity traded. Is the country a net exporter or importer? What are the consumer surplus, producer surplus, aggregate surplus, and gains from the trade?

**Solution:** The closed economy equilibrium is given by

\[Q^D(P) = 2000 - 100P = 100P = Q^S(P)\]
\[\Rightarrow \quad P^c = \frac{2000}{200} = 10\quad\text{and}\quad Q^c = 1000\]
\[\Rightarrow \quad AS = CS + PS = \frac{1}{2}(20 - 10)(1000) + \frac{1}{2}(10 - 0)(1000) = 10,000.\]
The open economy equilibrium is given by

\[ P^* = 5 \Rightarrow Q^S = 500 \text{ and } Q^D = 1500 \Rightarrow \text{net importer} \]

- \[ CS = \frac{1}{2}(20 - 5)(1500) = 11,250 \]
- \[ PS = \frac{1}{2}(5 - 0)(500) = 1250 \]
- \[ AS = 12,500 \text{ and gains from trade } = 2,500. \]

(b) Suppose the government wants to reduce the imports to 500 units by using tariffs. How should the government set the tariff to achieve this? Find the deadweight loss from the tariff.

**Solution:** We need

\[ Q^D(5 + t) - Q^S(5 + t) = 2000 - 100(5 + t) - 100(5 + t) = 500 \]
\[ \Rightarrow 2000 - 1000 - 200t = 500 \]
\[ \Rightarrow t = \frac{500}{200} = 2.5 \]

- \[ AS = \frac{1}{2}(20 - 7.5)(1250) + \frac{1}{2}(7.5)(750) + 2.5(500) \]
  \[ = 7812.5 + 2812.5 + 1250 = 11,875 \]

- \[ DWL = 12,500 - 11,875 = 625. \]

(c) Suppose there is a technological change in the domestic firms so that the domestic supply is now given by \( Q^S(P) = 400P \) while everything else remains the same. Find the new equilibrium price and the quantity traded. Is the country a net exporter or importer? What are the consumer surplus, producer surplus, aggregate surplus, and gains from the trade?

**Solution:** The closed economy equilibrium is now given by

\[ Q^D(P) = 2000 - 100P = 400P = Q^S(P) \]
\[ \Rightarrow P^c = \frac{2000}{500} = 4 \text{ and } Q^c = 1600 \]
\[ \Rightarrow AS = CS + PS = \frac{1}{2}(20 - 4)(1600) + \frac{1}{2}(4 - 0)(1600) = 12,800 + 3,200 \]
\[ = 16,000. \]

The open economy equilibrium is given by

\[ P^* = 5 \Rightarrow Q^S = 2000 \text{ and } Q^D = 1500 \Rightarrow \text{net exporter} \]

- \[ CS = \frac{1}{2}(20 - 5)(1500) = 11,250 \]
- \[ PS = \frac{1}{2}(5 - 0)(2000) = 5000 \]
- \[ AS = 16,250 \text{ and gains from trade } = 250. \]
8. Consider a market where the supply and the demand are given by

\[ Q^S(P) = 100P \quad \text{and} \quad Q^D(P) = 2000 - 100P. \]

(a) Find the equilibrium price, quantity, consumer surplus, producer surplus, and the aggregate surplus.

**Solution:**

\[ Q^S(P) = 100P = 2000 - 100P = Q^D(P) \]

\[ \Rightarrow P^* = 10 \quad \text{and} \quad Q^* = 100(10) = 1000. \]

The surpluses are

\[ CS = \frac{1}{2}(20 - 10)(1000) = 5000 \]
\[ PS = \frac{1}{2}(10 - 0)(1000) = 5000 \]
\[ AS = 10000. \]

(b) Suppose the government wants to raise revenue by imposing tax of ¥4 per unit. What is the price producers get, the price consumers pay, the equilibrium quantity, the tax revenue, and the dead weight loss?

**Solution:**

We now set

\[ Q^S(P) = 100(P^S) = 2000 - 100(P^S + t) = Q^D(P) \]

\[ 200P^S = 2000 - 100t \]

To obtain

\[ P^S = 10 - \frac{t}{2} = 8 \]
\[ P^D = 8 + 4 = 12 \]
\[ Q^T = 800 \]
\[ GR = 4(800) = 3200 \]
\[ DWL = (0.5)(t)(Q^* - Q^T) = (0.5)(4)(1000 - 800) = 400. \]

(c) Suppose the government is thinking about imposing an ad valorem tax instead of per unit tax. What does the tax rate have to be to keep the price consumers pay the same as in the per unit tax case?

**Solution:**

Setting

\[ Q^S(P) = 100(P^S) = 2000 - 100(P^S + tP^S) = Q^D(P) \]

yields

\[ 200P^S + 100tP^S = 2000 \quad \Rightarrow \quad P^S = \frac{2000}{200 + 100t} \]

We want

\[ (1 + t)P^S = 12 \quad \Rightarrow \quad (1 + t)\frac{2000}{200 + 50} = 12 \]

\[ \Rightarrow \quad 2000 + 2000t = 2400 + 1200t \]

\[ \Rightarrow \quad t = 50\% \]
We have

\[ p^S = \frac{2000}{200 + 100(0.5)} = 8 \]
\[ P^T = (1 + 0.5)p^S = 12 \]

(d) Which tax scheme is better for the economy? Why?

**Solution:** Note that even in valorem case, \( Q^T = 100p^S = 800 \). Since this means that \( CS, PS, GR, \) and \( DWL \) are all the same under both tax schemes. So they are equivalent.